The Mathematical Description of Colour

Ángel José Riesgo

Universidad de Oviedo

ariesgo@yahoo.com

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Introduction

Pixels: The building blocks of digital images

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In this talk I will be talking about:

- The geometric nature of colour
- The various colour models: RGB, HSL, CIELAB / CIELUV
- The standard CIE XYZ colour space
- Measuring differences between colours

Colour as a 3-dimensional quantity

The oldest colour model: names!

Name-based colour models

- Natural language (“red”, “green”, “červený”, “zelený”)
- Standardised codes based on a printed reference model. Example: Pantone.

Such colour models are obviously not appropriate for digital image processing.

Colours as three-dimensional vectors

- Colours are typically represented as points in a 3-dimensional compact space $C \subset \mathbb{R}^3$ (or $C \subset \mathbb{N}^3$).

Example: the RGB colour model

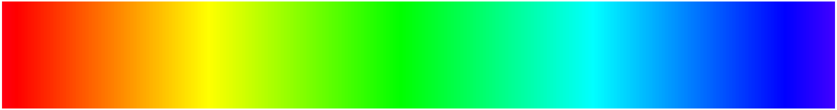
Why is colour three-dimensional?

Two explanations

- The physiology of the human eye: three types of cones.
- Intuitively, colour can be decomposed in three distinct elements.

And what are the three distinct elements that make up colour?

Let's begin with the spectral colours:

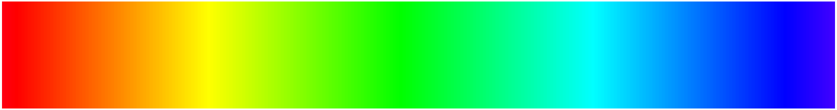


Spectral colours are defined by the wavelength of the light:
From red (380 nm) to violet (760 nm)[13].
This is a one-dimensional space $[380, 760]$ (or $[0, 1]$).

But it doesn't account for all the colours!

Where is brown or maroon?

Compare the following two spectra:



The difference is in the intensity of the light. We call this property **luminance** (also **luminosity**, **brightness** or **value**).

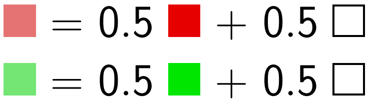
Fixed wavelength and variable luminance:

We now have a second dimension, but there are still missing colours.

Where is pink? ...Or white?

White light occurs when all the wavelengths are mixed:

Through "whitening" we can get additional colours:



Fixed wavelength and variable addition of white light:

$$\blacksquare = \lambda \, \blacksquare + (1 - \lambda) \, \square$$

This property is called **saturation** or **chroma** and is the third dimension of colour.

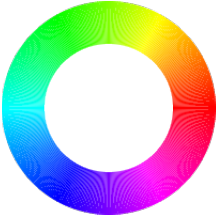
But whitened colours are not the only ones that mixing wavelengths produce.

By mixing red and violet, we get an additional range of quasi-spectral colours:

$$\blacksquare = \lambda \, \blacksquare + (1 - \lambda) \, \blacksquare$$

These are the **purples**.

So, the spectral colours can be complemented with the purples, giving rise to a **colour wheel**:



This way, we have completed the first dimension of colour, the **hue**.

When we express colour with the three values representing **hue**, **luminance** and **saturation**, we have a general form of the *HSL* colour model.

Such models are also often called **cylindrical** or **conical** models.

Colour models

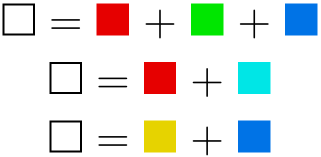
- Trichromatic models (RGB)
- Cylindrical (or conical) models (HSL, HSV, HSI, etc.)
- Luminance-bichromatic models (CIE LAB, CIE LUV)

There are various standard colour spaces for each one of these general models.

Mixing colours

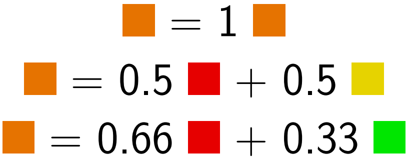
We saw that white light can result as a combination of many wavelength values:

But also:



Any given colour can be replicated through many alternative distributions of wavelengths.

For example, orange:



These are called **metamers**.

The existence of metamers indicates that colour is a psychological phenomenon, not easy to explain in purely physical terms.

Because of that, we cannot represent colour simply as a distribution of wavelength values plus luminance.

The 1931 CIE Colour Space

The 1931 CIE Colour Space

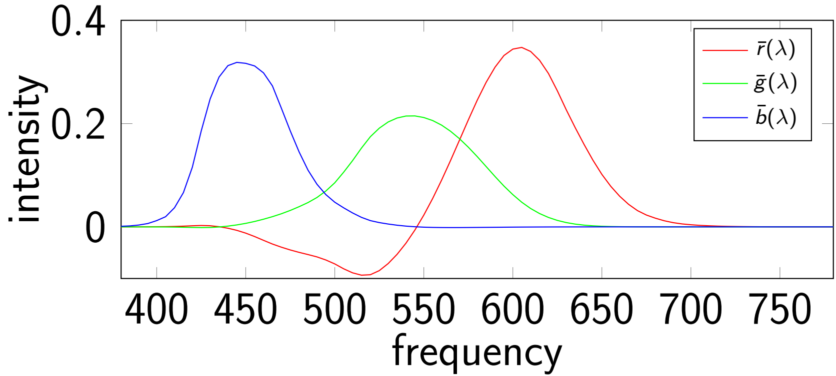
- Based on the experiments by D. Wright and J. Guild in the 1920's[24]
- Standardised in 1931 by the CIE (*Commission internationale de l'éclairage*)[3]

Wright and Guild's experimental setup

- Three lamps with primary monochromatic colours red, green and blue illuminating the same area.
- Additional lamp with variable monochromatic colour illuminating an adjacent area.
- A volunteer modifies red, green and blue intensities to match a fixed monochromatic colour.

Video with a good explanation of Wright and Guild's experiments[1]:
https://www.youtube.com/watch?v=KDiTxWcD3ZE

Wright and Guild's experiment results in the colour-matching functions $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, $\bar{b}(\lambda)$[7].



Negative values of $\bar{r}(\lambda)$ and $\bar{g}(\lambda) \implies$ Not all spectral colous reachable from the R, G, B primaries.

$\bar{r}(\lambda)$, $\bar{g}(\lambda)$, $\bar{b}(\lambda)$ + normalisation $\rightarrow$ **CIE RGB** space (trichromatic model with "imaginary colours")

Further normalisation conditions (white point) $\rightarrow$ **CIE XYZ** space (luminance-bichromatic model)

The X, Y (luminance), Z coordinates can additionally be transformed into:

$$x = \frac{X}{X+Y+Z}$$
$$y = \frac{Y}{X+Y+Z}$$
$$z = \frac{Z}{X+Y+Z}$$

These x, y, z coordinates have no units. It is common to use x and y together with Y for the luminance.

In the xyY space, if we fix the luminance Y we can represent the chromaticity on the xy plane. This is the chromaticity diagram.

Chromaticity diagram

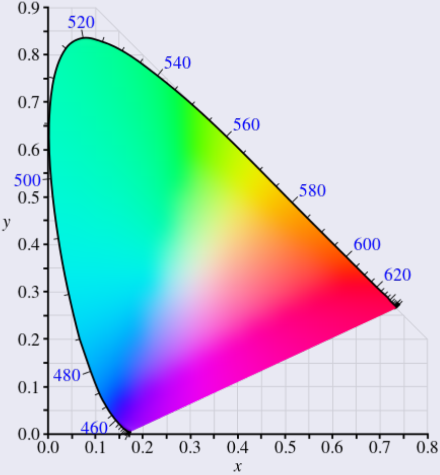
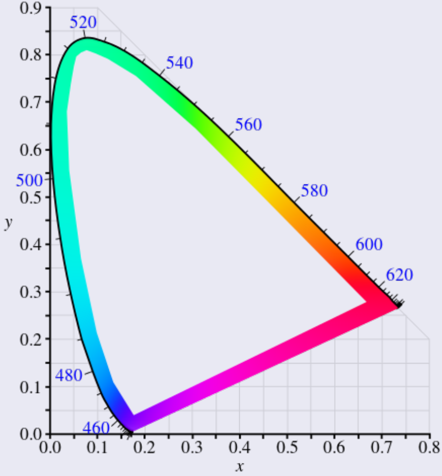


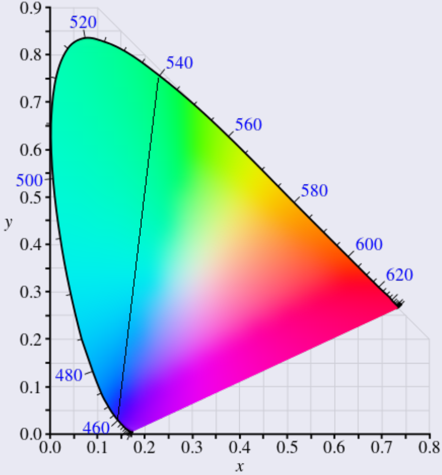
Image by user BenRG of Wikipedia, originally shared under a free licence (https://commons.wikimedia.org/wiki/File:CIE1931xy_blank.svg).

The chromaticity diagram gives us some very powerful insights into the geometric nature of colour.

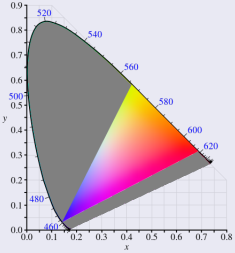
The hue as boundary

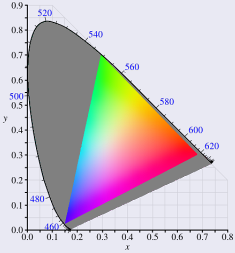


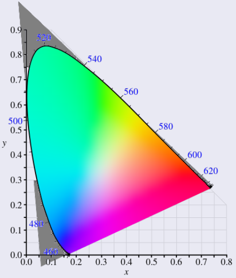
Colour mixes as linear combinations



The reach of trichromatic models







During the 20th century, the many device-dependent RGB colour spaces were defined in terms of the CIE XYZ space.

Today, the most widely used RGB colour space is the *sRGB* standard[12].

sRGB is specified in terms of CIE XYZ through a transformation matrix and γ correction (non-linear stretching of the luminance[11]).

Many cylindrical and conical colour spaces such as HSL, HSI, HSV, etc. have been defined based on simple geometric transformations of a reference RGB space.

They don't fully separate luminance, chroma and hue, and are problematic for digital image processing.

A better alternative is the **Improved HLS** (**IHLS**) colour space by Allan Hanbury and Jean Serra[6].

Measuring colour differences: the CIE LAB / CIE LUV colour spaces

Measuring the difference between two colours is very important in digital image processing (for example, when comparing the output of an algorithm with the desired result).

Can we simply use the Euclidean distance on an RGB-space?

As a broad approximation, yes. But there are better colour spaces.

The perceived difference in colours is a psychological phenomenon, so we need a transformation of space based on experimental data.

In 1942, David MacAdam carried out some colour perception tests with volunteers[9].

The result of these tests were the **MacAdam ellipses** on the chromaticity diagram.

Chromaticity diagram with MacAdam ellipses

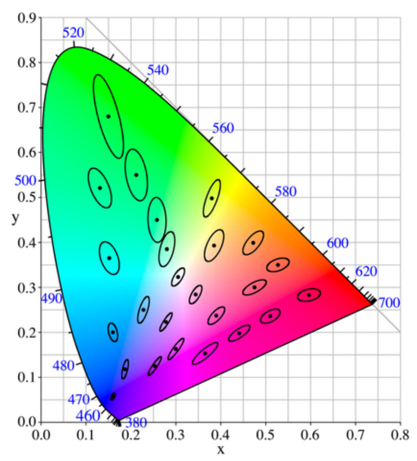


Image by user PAR commonswiki of Wikipedia, originally shared under a free licence. (https://commons.wikimedia.org/wiki/File:CIExy1931_MacAdam.png).

Mathematical problem

Define a coordinate transformation $XYZ \rightarrow X'Y'Z'$ such that the MacAdam ellipses become circles in the transformed space.

The problem is difficult. It cannot be solved while preserving some of the fundamental properties of the CIE XYZ space (like mixed colours as linear combinations, Euclidean geometry).

Two good approximations are the **CIE LAB** and **CIE LUV** colour spaces (1976)[14].

There are many software libraries that handle these colour space conversions. In MATLAB there are the rgb2lab and lab2rgb functions.

The problem of defining a good perceptual colour distance remains open[10].

Recent proposals

- CIEDE2000 Colour formula[8]
- J. Gravensen 2016[5]

Thank you!

[1] Craig Blackwell. Video on YouTube. https://www.youtube.com/watch?v=KDiTxWcD3ZE

[2] A. D. Broadbent. *Calculation from the original experimental data of the CIE 1931 RGB standard observer spectral chromaticity co-ordinates and color matching functions*. http://www.cis.rit.edu/research/mcsl2/research/broadbent/CIE1931_RGB.pdf

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[4] H. S. Fairman, M. H. Brill and H. Hemmendinger. *How the CIE 1931 colormatching functions were derived from the Wright-Guild data*. Color Res Appl 1997. Retrieved from http://infocom.uniroma1.it/~gaetano/texware/Full-How%20the%20CIE%201931%20Color-Matching%20Functions%20Were%20Derived%20from%20Wright-Guild%20Data.pdf.

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[6] Allen Hanbury, Jean Serra. *A 3D-polar coordinate colour representation suitable for image analysis*. Technical Report PRIP-TR-077, TU Wien 2002. http://cmm.ensmp.fr/~serra/notes_internes_pdf/NI-230.pdf

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[12] M. Stokes, M. Anderson, S. Chandrasekar and R. Motta. *A Standard Default Color Space for the Internet - sRGB* https://www.w3.org/Graphics/Color/sRGB

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